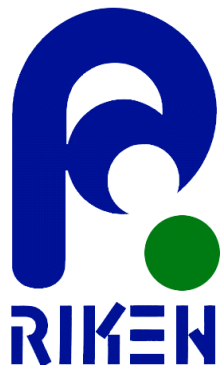


トポロジカル超流動体であるヘリウム3の 超流動相の理論研究



理研

堤 康雅

受賞対象論文

“Majorana Edge Modes of Superfluid ^3He A-Phase in a Slab”,
YT, T. Mizushima, M. Ichioka, and K. Machida, JPSJ **79**, 113601 (2010).

“Majorana surface states of superfluid ^3He A and B phases in a slab”,
YT, M. Ichioka, and K. Machida, PRB **83**, 094510 (2011).

超流動ヘリウム 3 の表面束縛状態

“Majorana Bound State in Rotating Superfluid ^3He -A between Parallel Plates”,
YT, T. Kawakami, T. Mizushima, M. Ichioka, and K. Machida,
PRL **101**, 135302 (2008).

超流動ヘリウム 3 の量子渦束縛状態

B相： YT, T. Kawakami, K. Shiozaki, M. Sato, and K. Machida, arXiv:1503.03136.

Outline

- トポロジカル超流動体・超伝導体

トポロジカル不変量

トポロジカル相転移

準粒子束縛状態

- 超流動ヘリウム3

A相の表面束縛状態と質量流

クーパー対の固有角運動量

B相の表面束縛状態とスピン流

トポロジカル超流動体・超伝導体

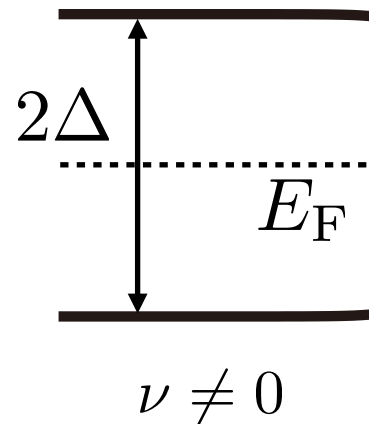
topological SF/SC	non-topological phase
topological invariant non zero	topological invariant zero

トポロジカル不変量：

ギャップのある系で定義される

トポロジカル相転移

準粒子束縛状態



トポロジカル不変量

例：2Dカイラル超流動状態

スピンレスBdG Hamiltonian

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \epsilon(\mathbf{k}) & \Delta(\mathbf{k}) \\ \Delta^*(\mathbf{k}) & -\epsilon(\mathbf{k}) \end{pmatrix} \quad \begin{aligned} \epsilon(\mathbf{k}) &= \hbar^2(k_x^2 + k_y^2 - k_F^2)/2m \\ \Delta(\mathbf{k}) &= (\Delta_0/k_F^l)(k_x + ik_y)^l = \Delta_0(k/k_F)^l e^{il\phi_k} \end{aligned}$$

固有値方程式

$$\begin{aligned} \mathcal{H}(\mathbf{k})|u_{\pm}(\mathbf{k})\rangle &= E_{\pm}(\mathbf{k})|u_{\pm}(\mathbf{k})\rangle \\ E_{\pm}(\mathbf{k}) &= \pm\sqrt{\epsilon(\mathbf{k})^2 + |\Delta(\mathbf{k})|^2} \\ |u_{\pm}(\mathbf{k})\rangle &= \begin{pmatrix} u_{\pm}(k)e^{il\phi_k/2} \\ v_{\pm}(k)e^{-il\phi_k/2} \end{pmatrix} \end{aligned}$$

ベリー接続 (ベクトルポテンシャル)

$$\mathbf{A}(\mathbf{k}) \equiv -i\langle u_{-}(\mathbf{k}) | \nabla_{\mathbf{k}} | u_{-}(\mathbf{k}) \rangle$$

First Chern number

$$\begin{aligned} \text{Ch}_1 &\equiv \frac{1}{2\pi} \int d\mathbf{k} [\nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})]_z \\ &= l \end{aligned}$$

トポロジカル不変量

例：2Dカイラル超流動状態

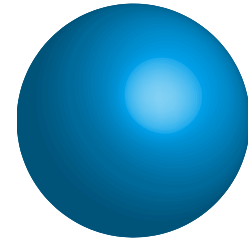
スピンレスBdG Hamiltonian

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \epsilon(\mathbf{k}) & \Delta(\mathbf{k}) \\ \Delta^*(\mathbf{k}) & -\epsilon(\mathbf{k}) \end{pmatrix} \quad \begin{aligned} \epsilon(\mathbf{k}) &= \hbar^2(k_x^2 + k_y^2 - k_F^2)/2m \\ \Delta(\mathbf{k}) &= (\Delta_0/k_F^l)(k_x + ik_y)^l = \Delta_0(k/k_F)^l e^{il\phi_k} \end{aligned}$$

2次元巻き付き数

$$w_{2D} = \text{Ch}_1$$

$$\mathcal{H}(\mathbf{k}) = \mathbf{m}(\mathbf{k}) \cdot \boldsymbol{\sigma} \quad \mathbf{m}(\mathbf{k}) = (\text{Re}[\Delta(\mathbf{k})], \text{Im}[\Delta(\mathbf{k})], \epsilon(\mathbf{k}))$$



s波

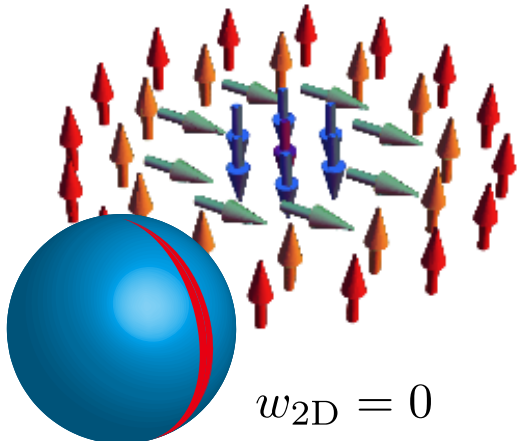
$$\mathbf{m}(\mathbf{k}) = (\Delta_0, 0, \epsilon(\mathbf{k}))$$

2Dカイラルp波

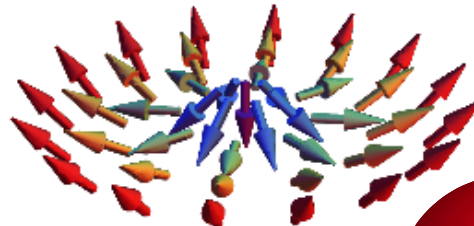
$$\mathbf{m}(\mathbf{k}) = (\Delta_0/k_F)(k_x, k_y, \epsilon'(\mathbf{k}))$$

2Dカイラルd波

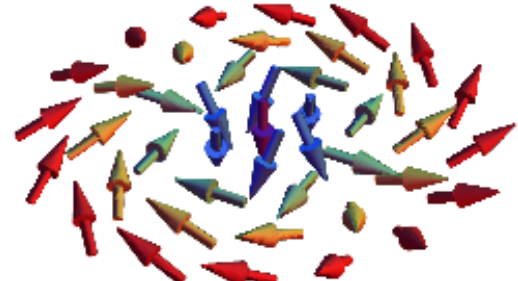
$$\mathbf{m}(\mathbf{k}) = (\Delta_0/k_F^2)(k_x^2 - k_y^2, 2k_x k_y, \epsilon''(\mathbf{k}))$$



$$w_{2D} = 0$$



$$w_{2D} = 1$$



$$w_{2D} = 2$$

トポロジカル超流動体・超伝導体

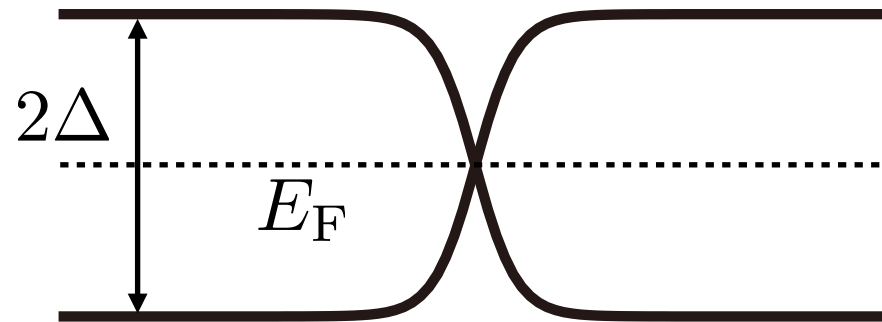
topological SF/SC	non-topological phase
topological invariant non zero	topological invariant zero

トポロジカル不変量：

ギャップのある系で定義される

トポロジカル相転移

準粒子束縛状態



$$\text{Ch}_1 = w_{2D} = 1$$

2Dカイラルp波

$$\text{Ch}_1 = w_{2D} = 0$$

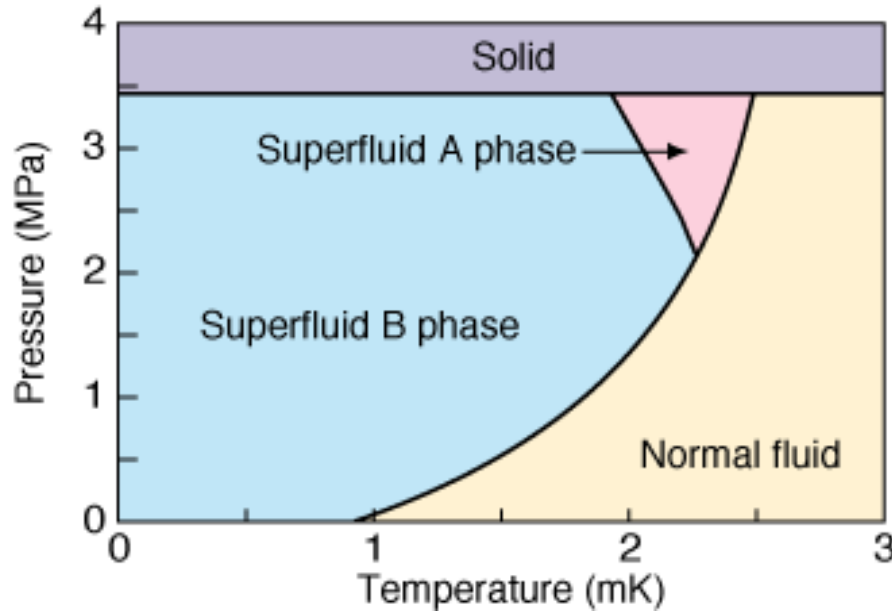
s波, 絶縁体、金属

表面束縛状態

超流動ヘリウム 3

^3He 原子のCooper対

スピン三重項p波状態



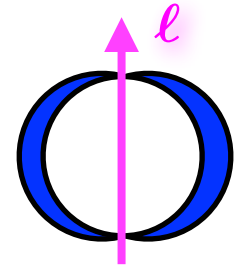
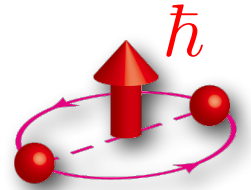
A-phase 高温、高圧領域

Anderson-Brinkman-Morel (ABM) 状態

スピン状態 $S_z = 0$ d

軌道状態 $L_z = 1$ ℓ

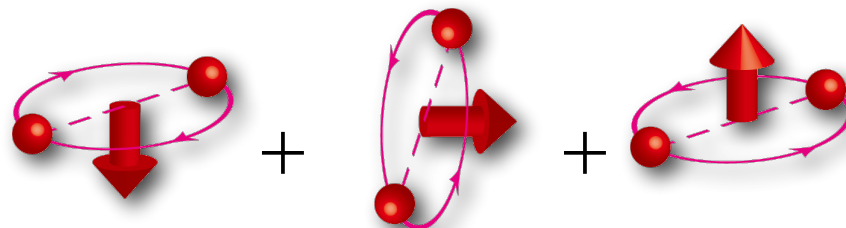
時間反転対称性の破れた状態



$$\begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_A}{k_F} \begin{pmatrix} k_x + ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix} \text{ポイントノード}$$

B-phase Balian-Werthamer (BW) 状態

$$L_z = -1 \quad L_z = 0 \quad L_z = +1$$



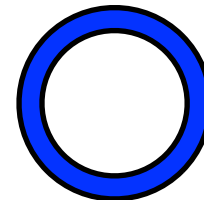
$$S_z = +1 \quad S_z = 0 \quad S_z = -1$$

$$\mathbf{J} = \mathbf{S} + \mathbf{L} = 0$$

時間反転対称

$$\begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_B}{k_F} \begin{pmatrix} -k_x + ik_y & k_z \\ k_z & k_x + ik_y \end{pmatrix}$$

等方的: full gap



超流動ヘリウム 3

^3He 原子のCooper対

スピン三重項p波状態

	TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
A (unitary)	0	0	0	-	Z	-
AI (orthogonal)	+1	0	0	-	-	-
AII (symplectic)	-1	0	0	-	Z_2	Z_2
AIII (chiral unitary)	0	0	1	Z	-	Z
BDI (chiral orthogonal)	+1	+1	1	Z	-	-
CII (chiral symplectic)	-1	-1	1	Z	-	Z_2
A-phase D	0	+1	0	Z_2	Z	-
C	0	-1	0	-	Z	-
B-phase DIII	-1	+1	1	Z_2	Z_2	Z
CI	+1	-1	1	-	-	Z

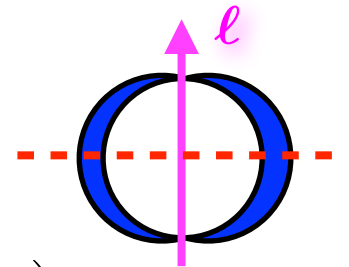
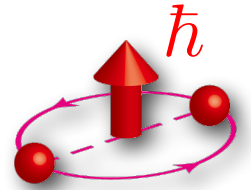
A-phase 高温、高圧領域

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スピン状態 $S_z = 0$ d

軌道状態 $L_z = 1$ ℓ

時間反転対称性の破れた状態

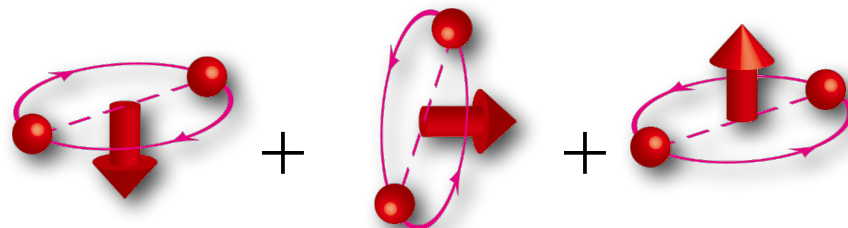


$$\begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_A}{k_F} \begin{pmatrix} k_x + ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix} \text{ポイントノード}$$

Schnyder *et al.*, PRB **78**, 195125 (2008).

B-phase Balian-Werthamer (BW) 状態

$$L_z = -1 \quad L_z = 0 \quad L_z = +1$$



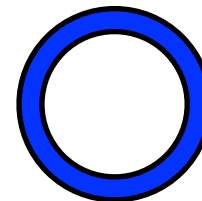
$$S_z = +1 \quad S_z = 0 \quad S_z = -1$$

$$\mathbf{J} = \mathbf{S} + \mathbf{L} = 0$$

時間反転対称

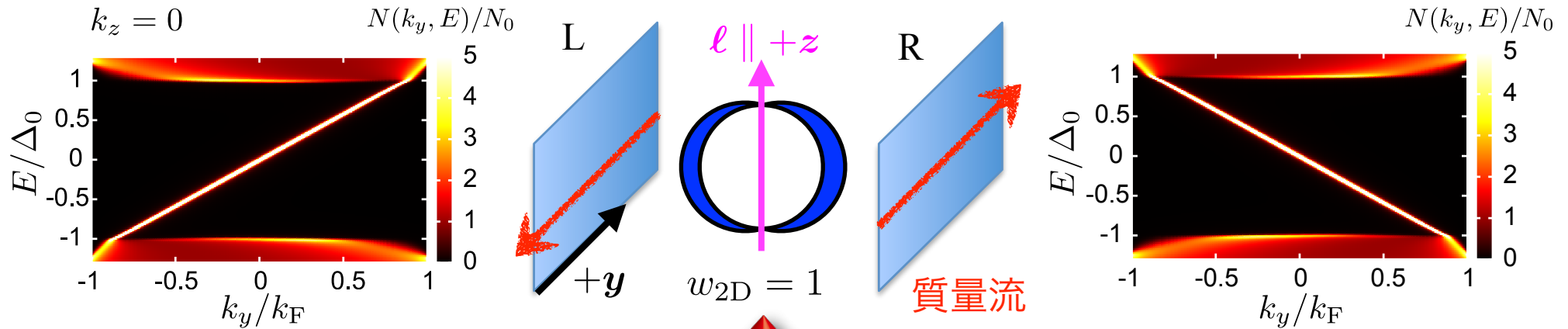
$$\begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\downarrow\uparrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} = \frac{\Delta_B}{k_F} \begin{pmatrix} -k_x + ik_y & k_z \\ k_z & k_x + ik_y \end{pmatrix}$$

等方的: full gap

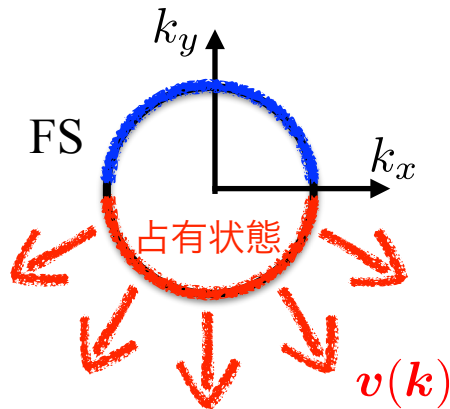


A相:表面束縛状態と質量流

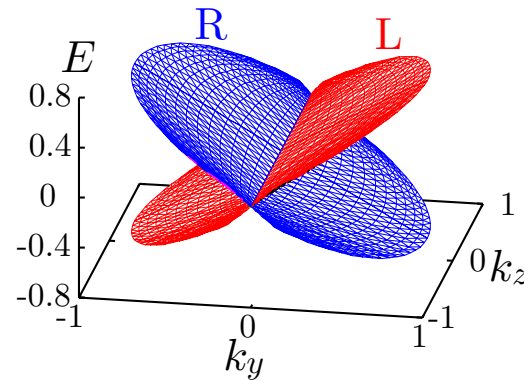
A相



$$\mathbf{v}(\mathbf{k}) = \frac{1}{\hbar} \nabla_{\mathbf{k}} E(\mathbf{k}) \parallel +\mathbf{y}$$



$$\langle \mathbf{v}(\mathbf{k}) \rangle_{\mathbf{k}} \parallel -\mathbf{y}$$

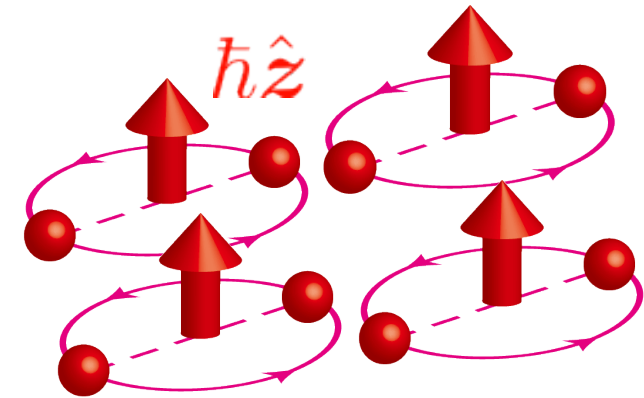


YT *et al.*, PRB **83**, 094510 (2011).

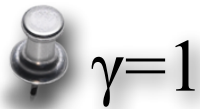
固有角運動量の問題

$$L_z \sim N\hbar \left(\frac{\Delta}{E_F} \right)^\gamma \quad \gamma = 0, 1, 2 ?$$

$$\Delta/E_F \sim 10^{-3}$$



N個の³He原子から成るA相の
バルクでの角運動量はいくらか？



$\gamma=1$

密度-カレント相関関数

$$\langle n(\mathbf{r}_1) \mathbf{j}(\mathbf{r}_2) \rangle \longrightarrow L_z \sim N\hbar \frac{\Delta}{E_F}$$

$$\text{凝縮粒子数} : N_c \sim N \frac{\Delta}{E_F}$$

Anderson and Morel, PR **123**, 1911 (1961).



$\gamma=2$

$$\mathbf{j} = k_B T \sum_n \int \frac{d\mathbf{k}}{(2\pi)^3} \mathbf{p} G_{\omega_n}(\mathbf{k})$$

gradient expansion
 $(k_F \xi)^{-1} \sim \Delta/E_F$

$$L_z \sim N\hbar \left(\frac{\Delta}{E_F} \right)^2$$

M. C. Cross, JLTP **21**, 525 (1975).



$\gamma=0$

N粒子BCS基底状態

$$|N\rangle = \frac{1}{A_N} \left[\int d\mathbf{r}_1 d\mathbf{r}_2 \phi(\mathbf{r}_2, \mathbf{r}_1) \psi^\dagger(\mathbf{r}_1) \psi^\dagger(\mathbf{r}_2) \right]^{(N/2)} |0\rangle$$

$$\langle N | \hat{L}_z | N \rangle \longrightarrow L_z = \frac{1}{2} N \hbar$$

相対運動部分

N/2個のクーパー対が角運動量 \hbar

M. Ishikawa, PTP **57**, 1836 (1977).

M. Ishikawa, K. Miyake, and T. Usui, PTP **63**, 1083 (1980).

準古典Eilenberger理論

$$\Delta/E_F \ll 1 \quad \int d\xi_k \hat{\sigma}_z \hat{G}(\mathbf{k}, \mathbf{r}, \omega_n) \equiv \hat{g}(\mathbf{k}_F, \mathbf{r}, \omega_n) \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\hat{f} & -\hat{g} \end{pmatrix}$$

Eilenberger方程式

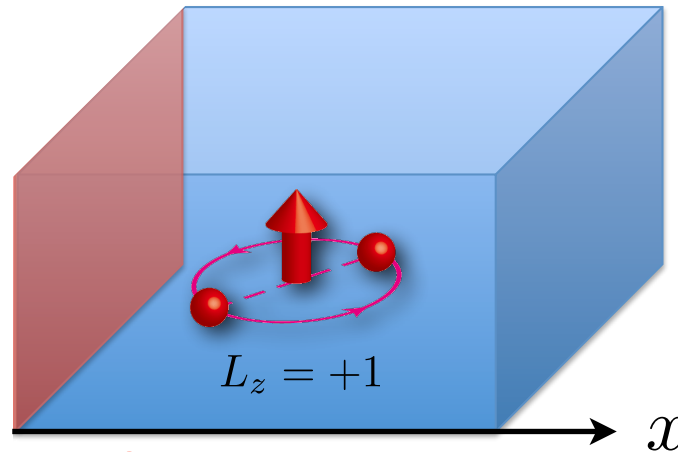
$$-i\hbar \mathbf{v}_F \cdot \nabla \hat{g}(\mathbf{k}_F, \mathbf{r}, \omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\mathbf{k}_F, \mathbf{r}) \\ \hat{\Delta}^\dagger(\mathbf{k}_F, \mathbf{r}) & -i\omega_n \hat{1} \end{pmatrix}, \hat{g}(\mathbf{k}_F, \mathbf{r}, \omega_n) \right]$$

$$\text{質量流 : } \mathbf{j}(\mathbf{r}, T) = N_0 \pi k_B T \sum_n \int \frac{d\Omega_{\mathbf{k}_F}}{4\pi} \mathbf{p}_F g_0(\mathbf{k}_F, \mathbf{r}, \omega_n)$$

$$\text{局所状態密度 : } N(\mathbf{k}_F, \mathbf{r}, E) = N_0 \text{Re}[g_0(\mathbf{k}_F, \mathbf{r}, \omega_n)|_{i\omega_n \rightarrow E+i\eta}]$$

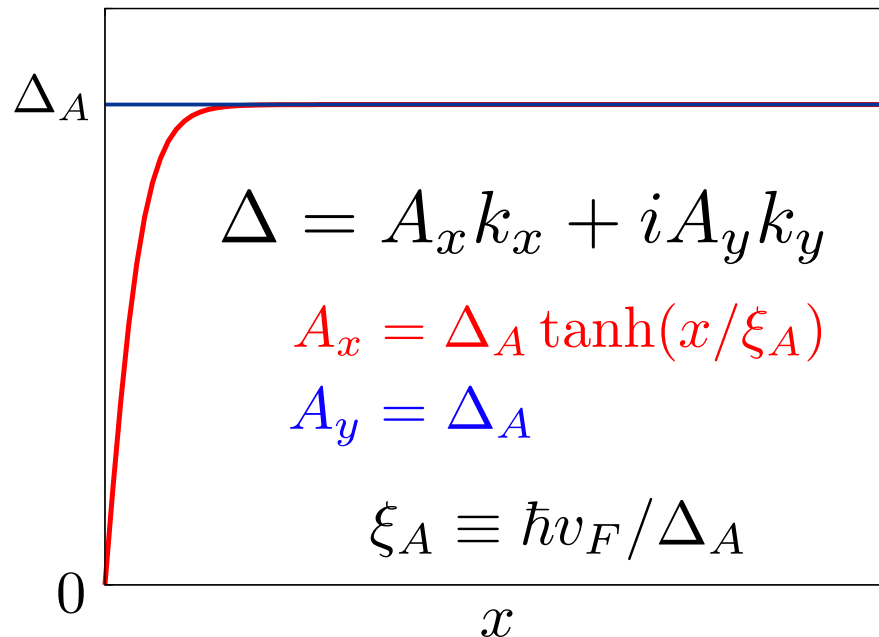
$$\text{質量流スペクトル : } \mathbf{j}(\mathbf{k}_F, \mathbf{r}, E) = \mathbf{p}_F N_0 \text{Re}[g_0(\mathbf{k}_F, \mathbf{r}, \omega_n)|_{i\omega_n \rightarrow E+i\eta}]$$

準古典グリーン関数



specular surface
 $x = 0$

Order parameter



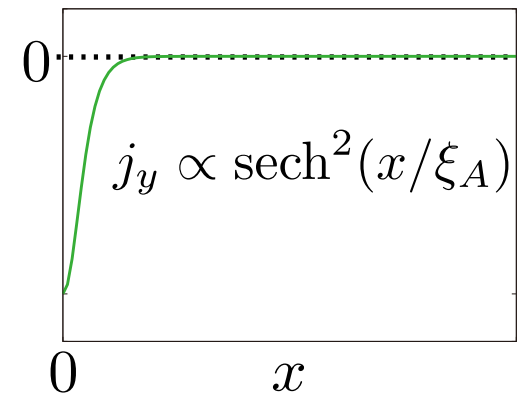
analytic solution: $(k_x = \cos \phi \sin \theta, k_y = \sin \phi \sin \theta)$

$$g_0 = \frac{1}{\sqrt{\omega_n^2 + \Delta_A^2 \sin^2 \theta}} \left[\omega_n + \frac{\Delta_A^2 \sin^2 \theta \cos^2 \phi}{2(\omega_n + i \Delta_A \sin \theta \sin \phi)} \operatorname{sech}^2 \left(\frac{x}{\xi_A} \right) \right]$$

bulk

Mass current

surface state



LDOS and current spectrum

$$N(0, \theta, E) \equiv \int \frac{d\phi}{2\pi} N(0, \mathbf{k}, E)$$



LDOS

bound state: $|E| < \Delta_A \sin \theta$

$$N(0, \theta, E) = \frac{N_0}{2} \quad \text{constant}$$

continuum state: $|E| > \Delta_A \sin \theta$

$$N(0, \theta, E) = \frac{N_0}{2} \left(\frac{|E|}{\sqrt{E^2 - \Delta_A^2 \sin^2 \theta}} + 1 \right)$$

surface state

bulk



Mass current

bound state:

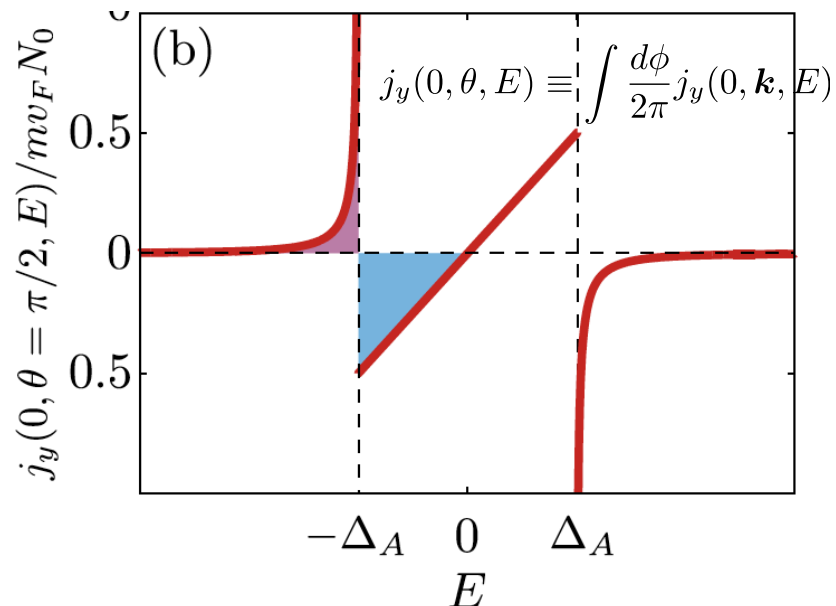
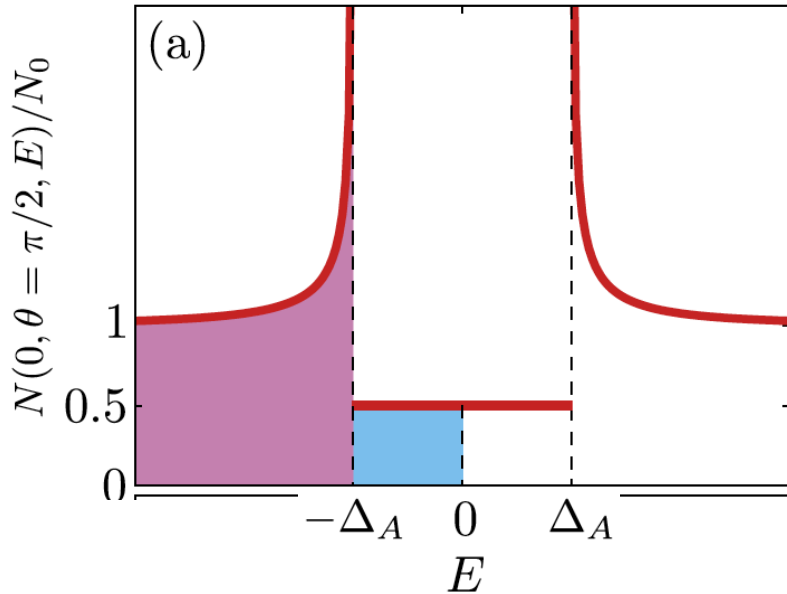
$$j_y(0, \theta, E) = \frac{mv_F N_0}{2} \frac{E}{\Delta_A} \quad \text{E-linear}$$

continuum state:

$$j_y(0, \theta, E) = -\frac{mv_F N_0}{4} \left(\text{sgn}(E) \frac{\sqrt{E^2 - \Delta_A^2 \sin^2 \theta}}{\Delta_A} + \frac{E|E|}{\Delta_A \sqrt{E^2 - \Delta_A^2 \sin^2 \theta}} - 2 \frac{E}{\Delta_A} \right)$$

$$\approx -\frac{mv_F N_0}{16} \sin \theta \left(\frac{\Delta_A \sin \theta}{E} \right)^3 \quad (|E| \gg \Delta_A \sin \theta)$$

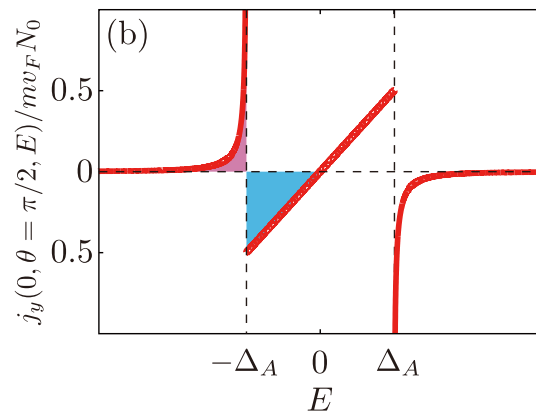
ギャップから離れると E^{-3} で減衰.



Total current and angular momentum

Zero temperature

 **Total mass current**



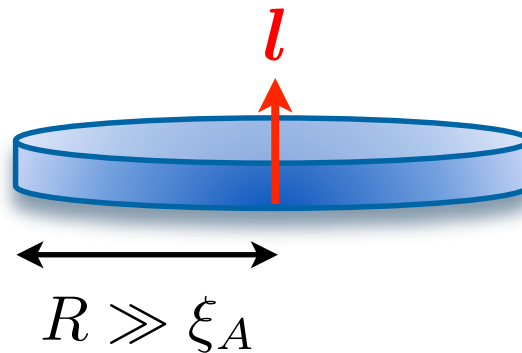
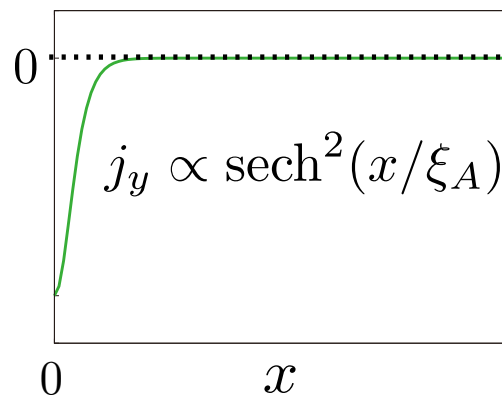
$$J_y^{\text{bound}} = \int_0^\infty dx \left\langle \int_{-\Delta_A \sin \theta}^0 dE j_y(x, \mathbf{k}, E) \right\rangle_{\mathbf{k}} = -\frac{n\hbar}{2}$$

$$J_y^{\text{cont}} = \int_0^\infty dx \left\langle \int_{-\infty}^{-\Delta_A \sin \theta} dE j_y(x, \mathbf{k}, E) \right\rangle_{\mathbf{k}} = \frac{n\hbar}{4}$$

$$\left(N_0 = \frac{3}{mv_F^2} n \right)$$

$$J_y = J_y^{\text{bound}} + J_y^{\text{cont}} = -\frac{n\hbar}{4}$$

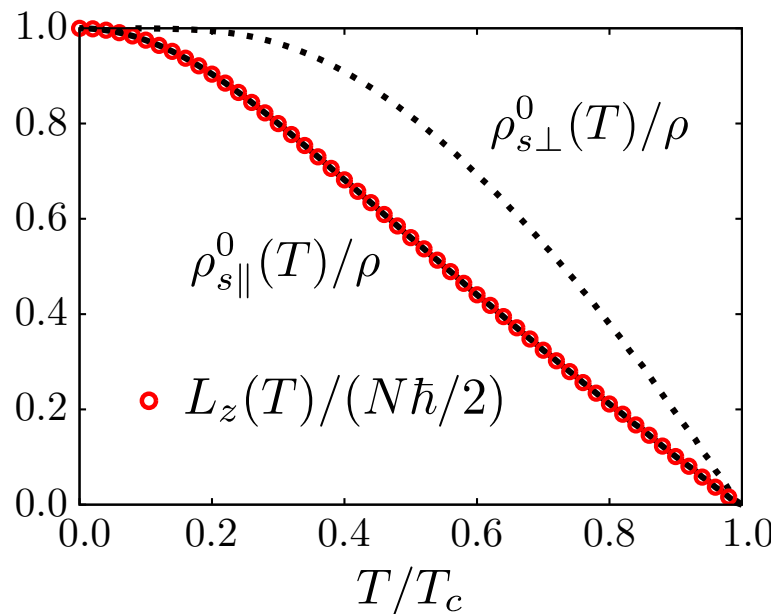
 **Angular momentum**



$$L_z = \frac{N\hbar}{2}$$

(N : number of ^3He atoms)

Temperature dependence

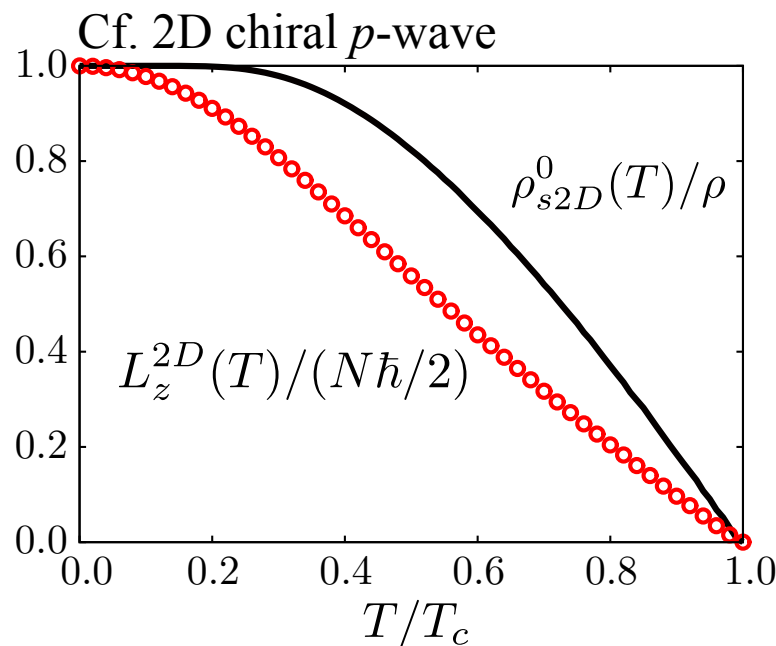


$$L_z(T)/(N\hbar/2) = \rho_{s\parallel}^0(T)/\rho$$

偶然？

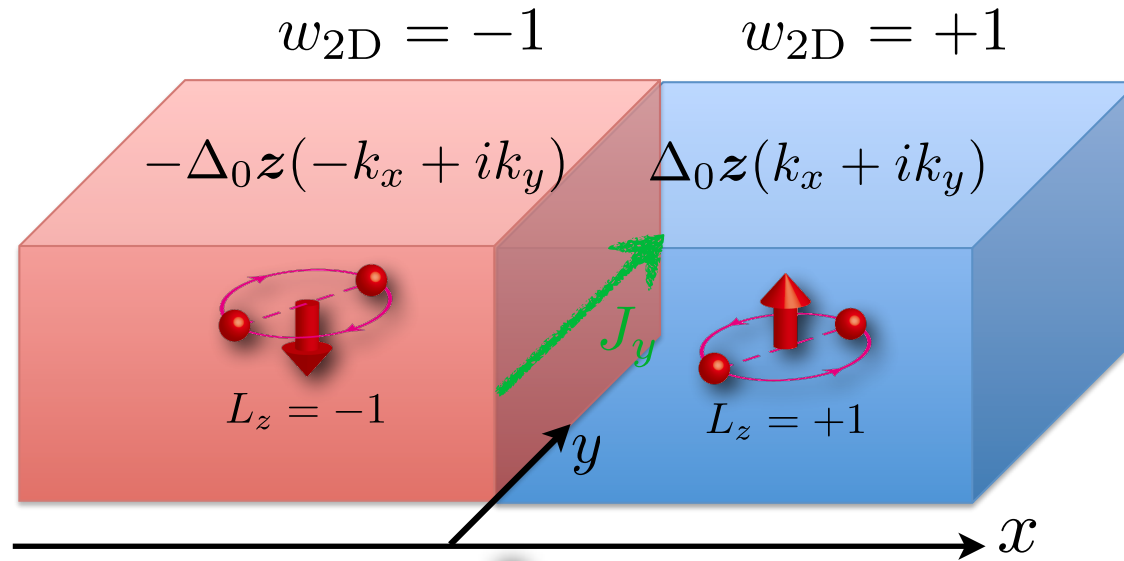
consistent with T. Kita, JPSJ **67**, 216 (1998).

$$L_z(T) = \frac{3}{4} N \hbar \frac{\pi k_B T}{\Delta_A} \sum_{\omega_n} \left[\frac{3\omega_n^2 + \Delta_A^2}{\Delta_A^2} \sin^{-1} \frac{\Delta_A}{\sqrt{\omega_n^2 + \Delta_A^2}} - 3 \frac{|\omega_n|}{\Delta_A} \right]$$

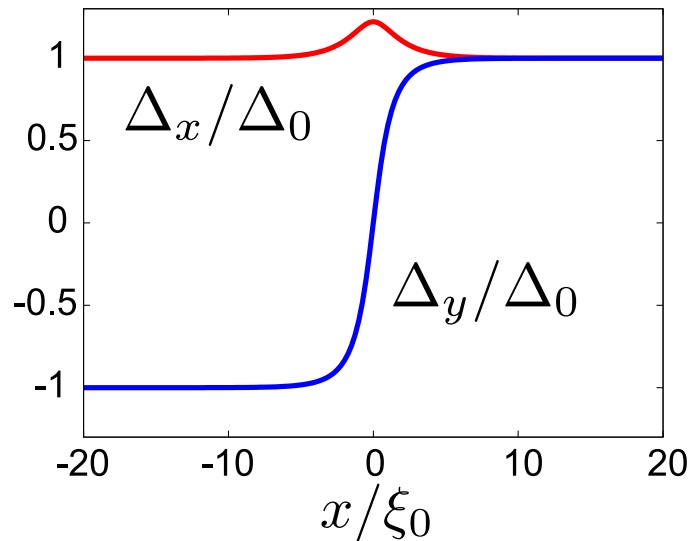


Y. Nagato, S. Higashitani, and K. Nagai, JPSJ **80**, 113706 (2011).
J.A. Sauls, PRB **84**, 214509 (2011).

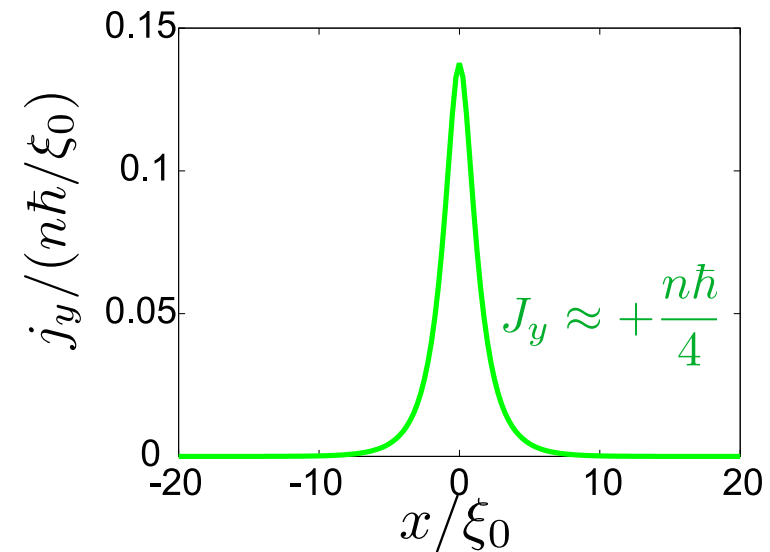
Chiral domain wall



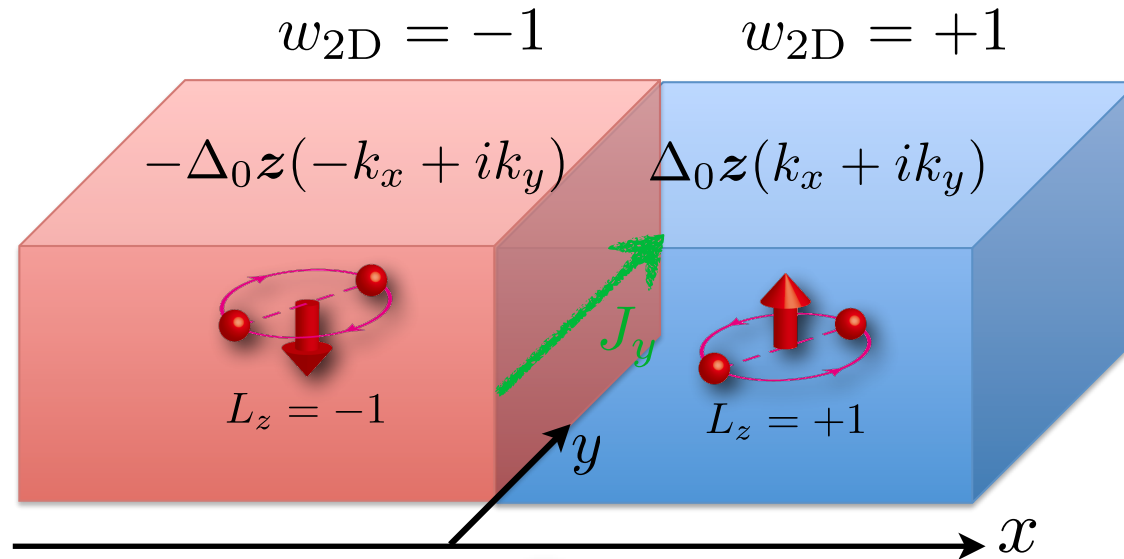
 **order parameter**



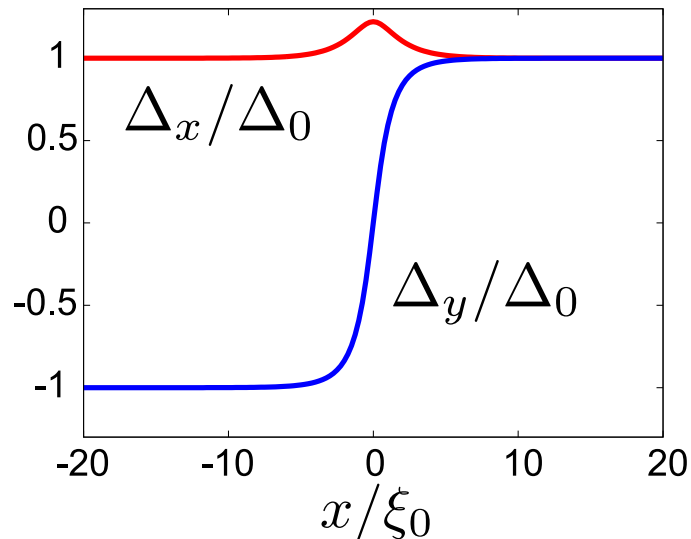
 **mass current**



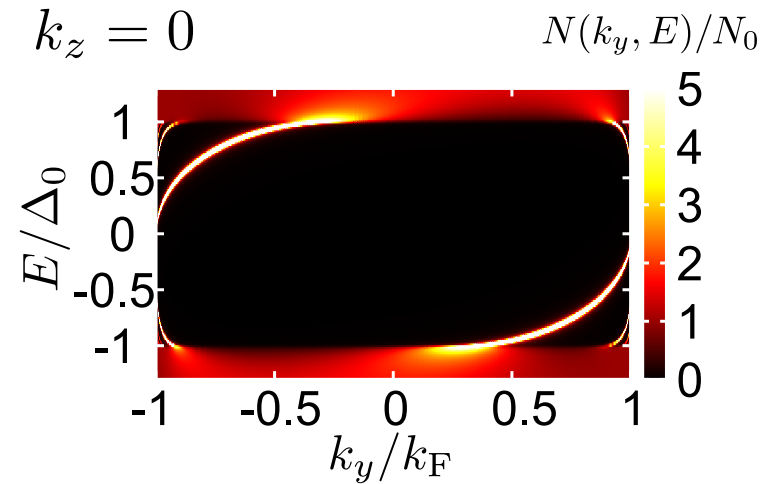
Chiral domain wall



order parameter

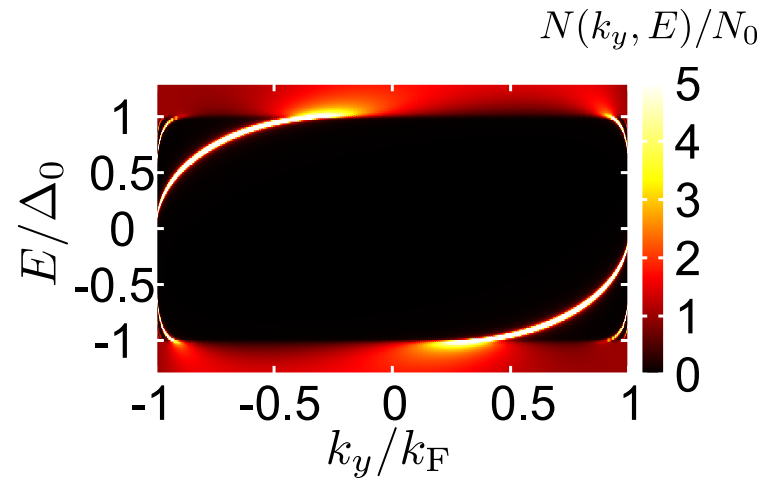
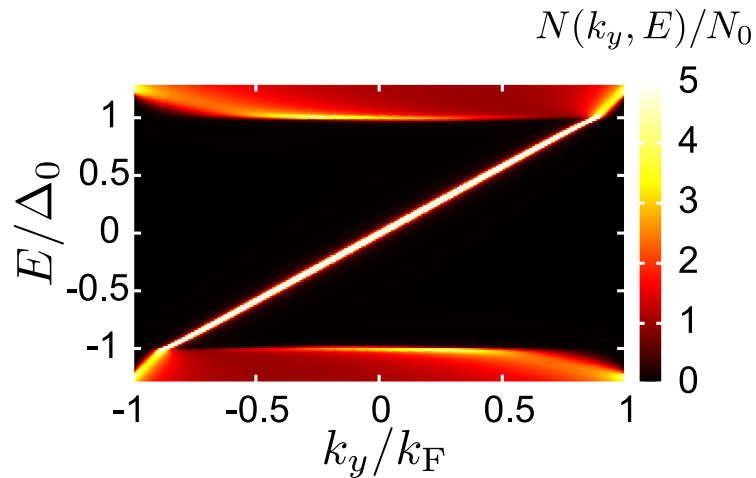
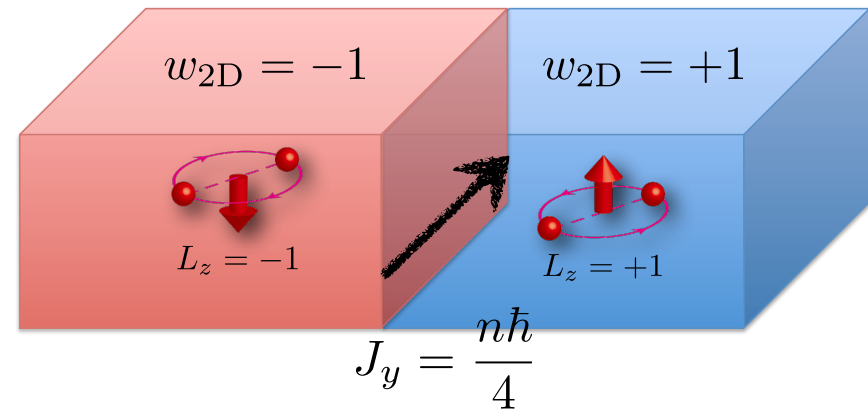
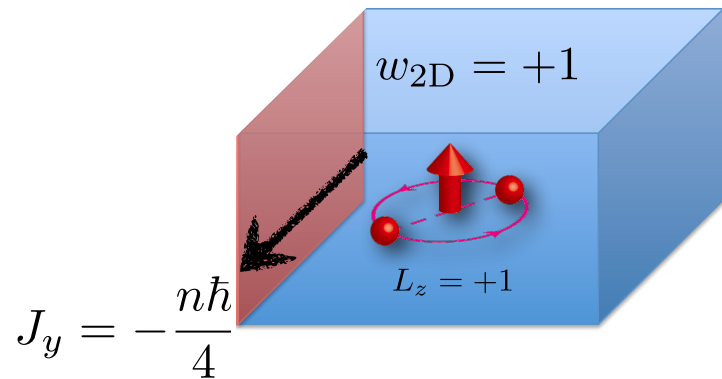


dispersion at domain wall



$$\Delta w_{2D} = 2$$

Spectral flow



2Dカイラル ℓ 波

$$k_y/k_F = \cos \left[\left(n - \frac{1}{2} \right) \frac{\pi}{l} \right], \quad n = 1, 2, \dots, l$$

$$\Delta J_y = l \frac{n\hbar}{4}, \quad (l \geq 2)$$

microscopic discussion

See Y. Tada *et al.*, arXiv:1409.7459.

$$\begin{aligned} \Delta J_y &= \frac{n\hbar}{2} \sum_{E=0} \left(\frac{k_y}{k_F} \right)^2 \text{sgn} \left(\frac{\partial E}{\partial k_y} \right) \\ &= \frac{n\hbar}{2} \end{aligned}$$

G. E. Volovik, Pis'ma ZhETF **66**, 492 (1997).

バルク角運動量

A. Shitade and T. Kimura, PRB 90, 134510 (2014).

2Dカイラルp波

$$L_z = -\hbar \sum_{\mathbf{k}} [\mathbf{A}(\mathbf{k}) \times \mathbf{k}]_z = \frac{N\hbar}{2}$$

ベリー接続

$$\mathbf{A}(\mathbf{k}) \equiv -i \langle u_-(\mathbf{k}) | \nabla_{\mathbf{k}} | u_-(\mathbf{k}) \rangle$$

ポイントノードのあるA相ではベリー接続を定義できない

B相:表面束縛状態とスピン流

トポロジカル不変量 = 3次元巻き付き数

$$w_{3D}$$

BdG Hamiltonian

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \epsilon(\mathbf{k}) & i(\Delta_0/k_F)\mathbf{k} \cdot \boldsymbol{\sigma} \sigma_y \\ -i(\Delta_0/k_F)\sigma_y \mathbf{k} \cdot \boldsymbol{\sigma} & -\epsilon(\mathbf{k}) \end{pmatrix}$$

$$= \mathbf{m}(\mathbf{k}) \cdot \boldsymbol{\gamma}$$

$$\gamma_1 = \begin{pmatrix} 0 & -\sigma_z \\ -\sigma_z & 0 \end{pmatrix} \quad \gamma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

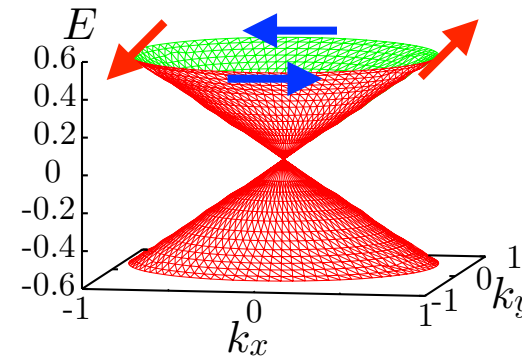
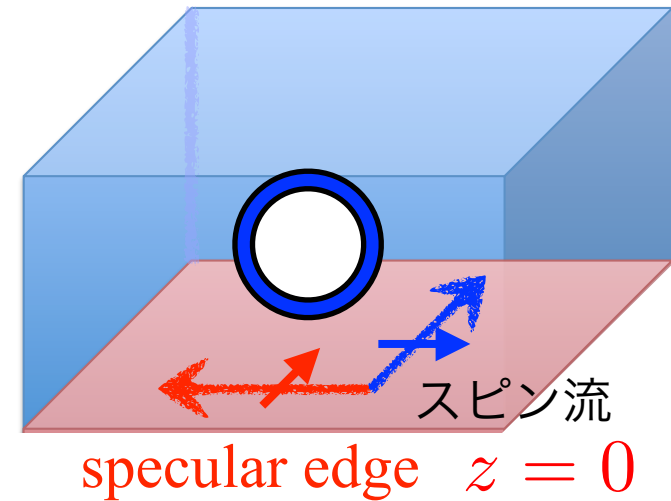
$$\gamma_3 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix} \quad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{m}(\mathbf{k}) = (\Delta_0/k_F)(k_x, k_y, k_z, \epsilon'(\mathbf{k}))$$

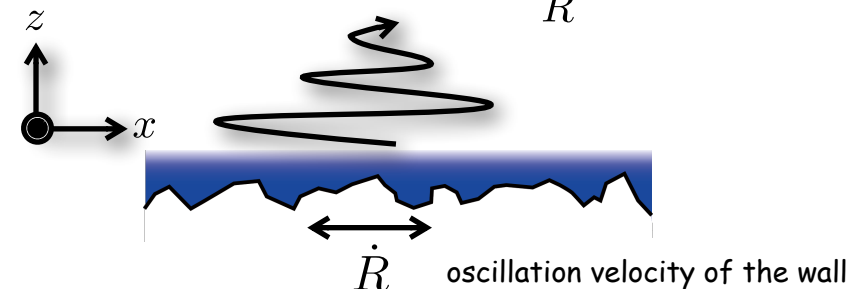
$$w_{3D} = 1$$

Cf. 2Dカイラルp波

$$\mathbf{m}(\mathbf{k}) = (\Delta_0/k_F)(k_x, k_y, \epsilon'(\mathbf{k}))$$



acoustic impedance $Z' + iZ'' = \frac{\Pi_{xz}}{\dot{R}}$ stress tensor of 3He



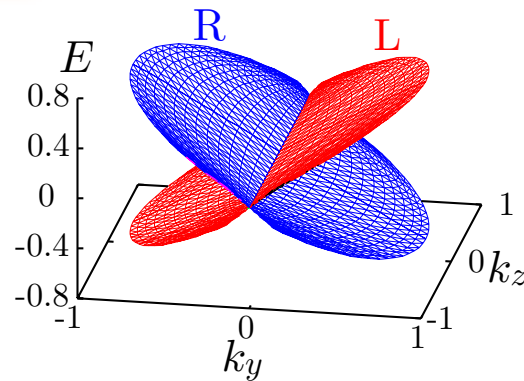
Y. Okuda and R. Nomura,
J. Phys.: Condens. Matter **24**, 343201 (2012).

Summary

- トポロジカル超流動体であるヘリウム3の表面束縛状態と表面カレントを研究

A相

$w_{2D} = 1$ フラットな線形分散
表面質量流



T=0での角運動量

有限系（回転対称） バルク

2D

$$N\hbar/2$$

$$N\hbar/2$$

3D

$$N\hbar/2$$

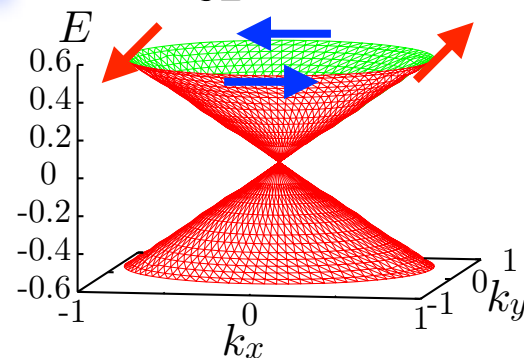
$$N\hbar/2$$

$$N\hbar(\Delta/E_F)^2 ?$$

表面質量流による
角運動量

B相

$w_{3D} = 1$



コーン型分散
表面スピン流