2005年度冬学期 振動波動論 解答例

第１問

$$f(x)=\sum_{n=1}^{\infty} c_n \sin(n\pi x)$$

の両辺に $\sin(n\pi x)$ をかけて 0 から 1 まで積分すると

$$\begin{aligned} m\neq n \quad & \int_0^1 \sin(n\pi x)\sin(m\pi x)dx = 0 \\ m=n \quad & \int_0^1 \sin(n\pi x)\sin(m\pi x)dx = \frac{1}{2} \end{aligned}$$

を用いて

$$\begin{aligned} & \int_0^1 f(x)\sin(n\pi x)dx = \frac{1}{2}c_n \\ \Leftrightarrow \quad & c_n = 2\left[\int_0^{\frac{1}{4}} 4x\sin(n\pi x)dx + \int_{\frac{1}{4}}^1 \frac{4(1-x)}{3}\sin(n\pi x)dx\right] \end{aligned}$$

後述する関係式を用いると上式は

$$\begin{aligned} c_n = & \left\{\frac{8}{(n\pi)^2}\sin\left(\frac{n}{4}\pi\right) - \frac{2}{n\pi}\cos\left(\frac{n}{4}\pi\right)\right\} \\ + & \left\{-\frac{8}{3n\pi}\cos(n\pi) + \frac{8}{3n\pi}\cos\left(\frac{n}{4}\pi\right)\right\} \\ + & \left\{\frac{8}{3n\pi}\cos(n\pi) - \frac{2}{3n\pi}\cos\left(\frac{n}{4}\pi\right) + \frac{8}{3(n\pi)^2}\sin\left(\frac{n}{4}\pi\right)\right\} \\ = & \frac{32}{3(n\pi)^2}\sin\left(\frac{n}{4}\pi\right)\cdots(\text{答}) \end{aligned}$$

となる。ここで以下の三式を用いた。

$$\begin{aligned} \int_0^{\frac{1}{4}} x\sin(n\pi x)dx &= \left[-\frac{1}{n\pi}x\cos(n\pi x)\right]_0^{\frac{1}{4}} + \int_0^{\frac{1}{4}}\frac{1}{n\pi}\cos(n\pi x)dx \\ &= -\frac{1}{4n\pi}\cos\left(\frac{n}{4}\pi\right) + \left[\frac{1}{(n\pi)^2}\sin(n\pi x)\right]_0^{\frac{1}{4}} \\ &= \frac{1}{(n\pi)^2}\sin\left(\frac{n}{4}\pi\right) - \frac{1}{4n\pi}\cos\left(\frac{n}{4}\pi\right) \\ \int_{\frac{1}{4}}^1 x\sin(n\pi x)dx &= \left[-\frac{1}{n\pi}x\cos(n\pi x)\right]_{\frac{1}{4}}^1 + \int_{\frac{1}{4}}^1 \frac{1}{n\pi}\cos(n\pi x)dx \end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{n\pi}\cos(n\pi) + \frac{1}{4n\pi}\cos\left(\frac{n}{4}\pi\right) + \left[\frac{1}{(n\pi)^2}\sin(n\pi x)\right]_{\frac{1}{4}}^{1} \\
&= -\frac{1}{n\pi}\cos(n\pi) + \frac{1}{4n\pi}\cos\left(\frac{n}{4}\pi\right) - \frac{1}{(n\pi)^2}\sin\left(\frac{n}{4}\pi\right)
\end{aligned}
$$

$$
\begin{aligned}
\int_{\frac{1}{4}}^{1}\sin(n\pi x)dx &= \left[-\frac{1}{n\pi}\cos(n\pi x)\right]_{\frac{1}{4}}^{1} \\
&= -\frac{1}{n\pi}\cos(n\pi) + \frac{1}{n\pi}\cos\left(\frac{n}{4}\pi\right)
\end{aligned}
$$

元の曲線 $y = f(x)$ と $y = c_1 \sin(\pi x) = \frac{16\sqrt{2}}{3\pi^2}\sin(\pi x)$ とを図示したものが以下。

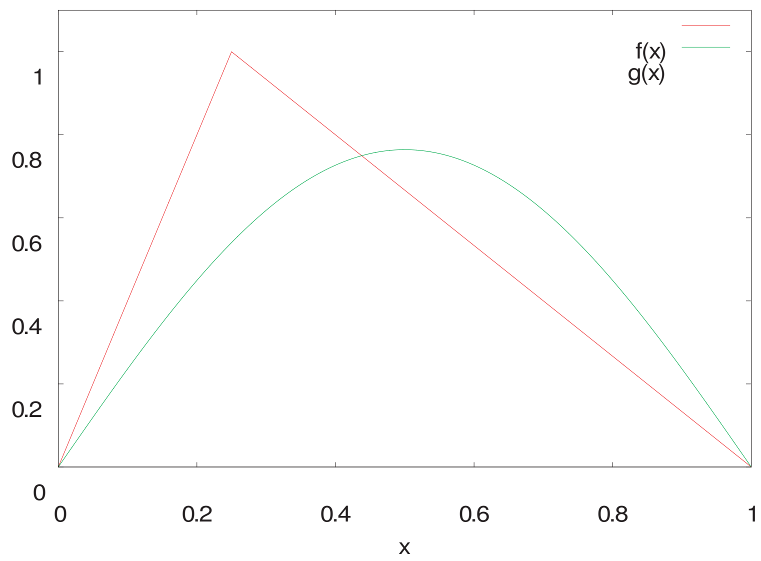

図 1: $y = f(x)$,$y = \frac{16\sqrt{2}}{3\pi^2}\sin(\pi x)$ のグラフ

元の曲線 $y=f(x)$ と $y=c_1\sin(\pi x)+c_2\sin(2\pi x)+c_3\sin(3\pi x)=\frac{16\sqrt{2}}{3\pi^2}\sin(\pi x)+\frac{8}{3\pi^2}\sin(2\pi x)+\frac{16\sqrt{2}}{27\pi^2}\sin(3\pi x)$ とを図示したものが以下。

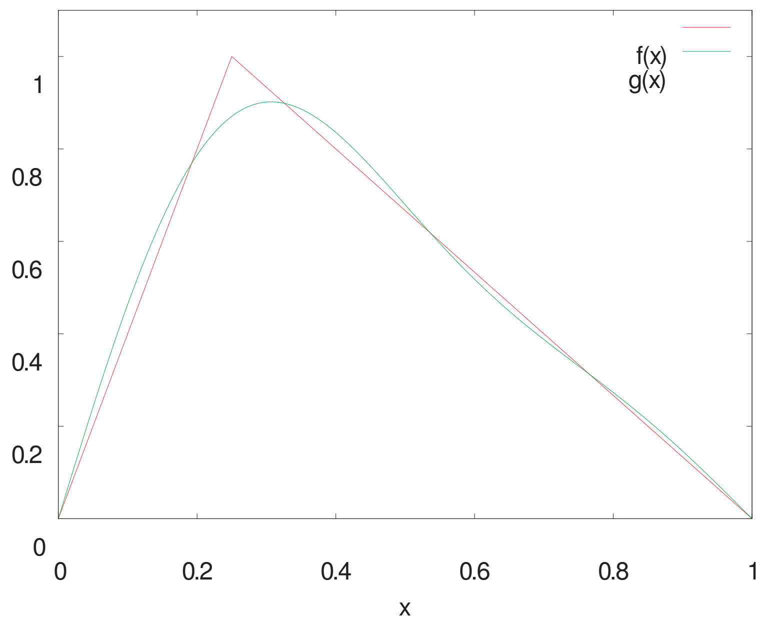

図 2: $y=f(x)$,$y=\frac{16\sqrt{2}}{3\pi^2}\sin(\pi x)+\frac{8}{3\pi^2}\sin(2\pi x)+\frac{16\sqrt{2}}{27\pi^2}\sin(3\pi x)$ のグラフ

元の曲線 $y = f(x)$ と $y = c_1 \sin(\pi x) + c_2 \sin(2\pi x) + c_3 \sin(3\pi x) + c_4 \sin(4\pi x) + c_5 \sin(5\pi x) = \frac{16\sqrt{2}}{3\pi^2} \sin(\pi x) + \frac{8}{3\pi^2} \sin(2\pi x) + \frac{16\sqrt{2}}{27\pi^2} \sin(3\pi x) - \frac{16\sqrt{2}}{75\pi^2} \sin(5\pi x)$ とを図示したものが以下。

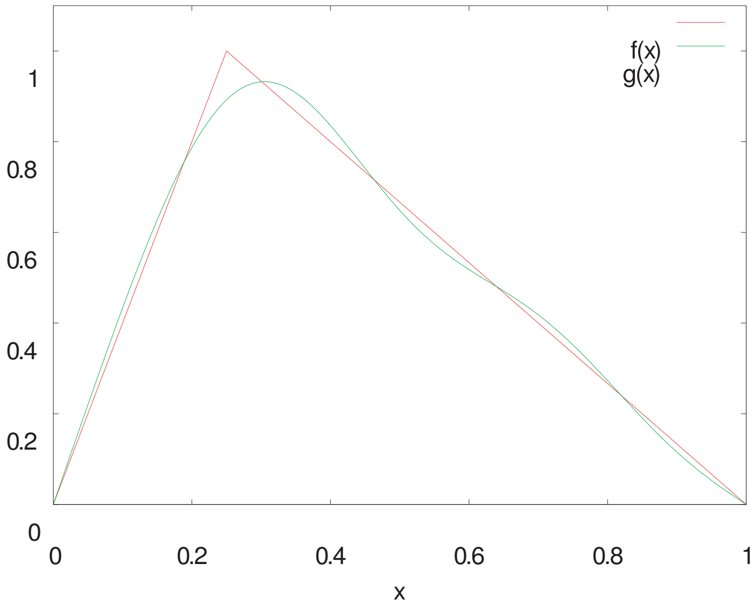

図 3: $y = f(x)$,$y = \frac{16\sqrt{2}}{3\pi^2} \sin(\pi x) + \frac{8}{3\pi^2} \sin(2\pi x) + \frac{16\sqrt{2}}{27\pi^2} \sin(3\pi x) - \frac{16\sqrt{2}}{75\pi^2} \sin(5\pi x)$ のグラフ